BAAO

British Astronomy and Astrophysics Olympiad

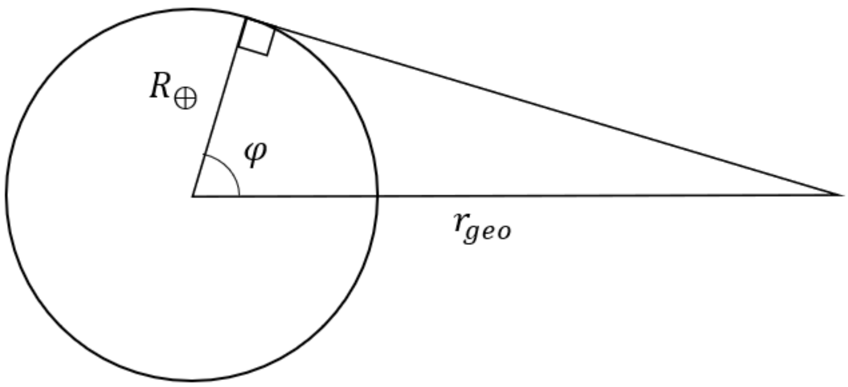
BAAO Astro Challenge

November / December 2023

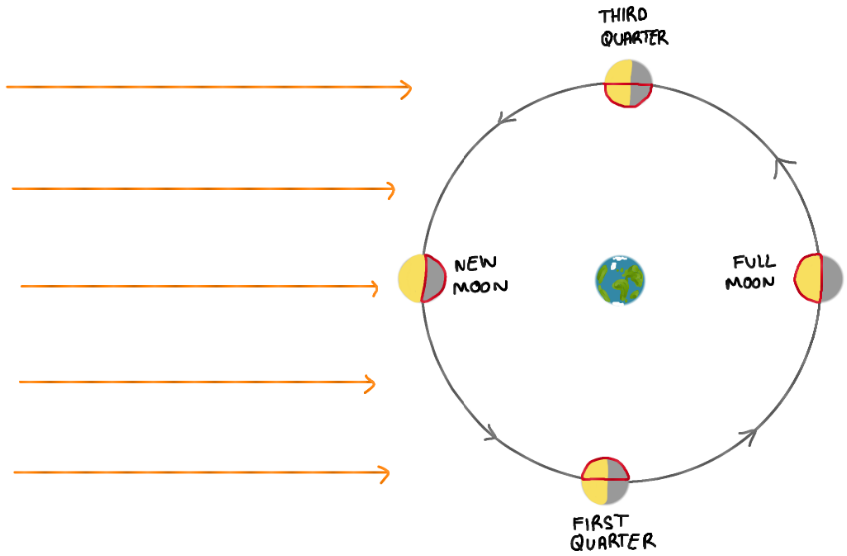
Solutions and marking guidelines

- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more than that, especially for intermediate stages, to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer. Students getting the answer in a $\boxed{\text{box}}$ will get all the marks available for that calculation / part of the question (students may not explicitly calculate the intermediate stages, and should not be penalised for this so long as their argument is clear)

Question	Answer	Mark
Section A		**10**
1.	D The other places are other current UK spaceports (like Spaceport Cornwall in Newquay, which had its first launch Jan 2023) or ones proposed to open in 2024 (Sutherland Spaceport and Spaceport Snowdonia). The Shetland Islands are uniquely well placed for a spaceport as their high latitude means they can launch a greater payload for a given amount of fuel, whilst also being remote enough for safety should there be any issues at launch	1
2.	D The period of an orbit is found with Kepler's 3rd Law: $$T = \sqrt{\frac{4\pi^2}{GM}r^3} \therefore T \propto r^{3/2}$$ The speed in a circular orbit is: $$v = \sqrt{\frac{GM}{r}} \therefore v \propto r^{-1/2}$$ To get the r to cancel each other out in the product we need Tv^3: $Tv^3 = 2\pi(GM)^{-1/2}r^{3/2} \times (GM)^{3/2}r^{-3/2} = 2\pi GM = \text{constant in system}$	1

3.	A We first convert 21 million light years into metres, and then into parsecs $$21 \text{ Mly} = 21 \times 10^6 \times \left((3 \times 10^8) \times (365 \times 24 \times 60 \times 60)\right)$$ $$= 1.99 \times 10^{23} \text{ m} = 6.43 \text{ Mpc}$$ We can then use the standard formula to convert from apparent to absolute magnitude $$\mathcal{M} = m - 5\log\left(\frac{d}{10}\right) = 10.8 - 5\log\left(\frac{6.43 \times 10^6}{10}\right) = -18.24$$	1
4.	B The distance from the Sun to the Moon is effectively the same as the distance from the Sun to Earth, and so $$b = \frac{L}{4\pi d^2} = \frac{L_\odot}{4\pi(1 \text{ au})^2} = \frac{3.83 \times 10^{26}}{4\pi(1.50 \times 10^{11})^2} = 1355 \text{ W m}^{-2}$$ Assuming the panel is always perpendicular to the sunlight $$\text{area} = \frac{P}{b} = \frac{50}{1355} = 0.0369 \text{ m}^2 = 369 \text{ cm}^2 \approx 370 \text{ cm}^2$$ (This is only a lower limit as the real panel is not 100% efficient and the movement of the rover means it does not always face directly at the Sun)	1
5.	C The radius of a geostationary orbit is $$r_{geo} = \sqrt[3]{\frac{GM_\oplus}{4\pi^2}T^2} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{4\pi^2} \times (24 \times 60 \times 60)^2}$$ $$= 4.22 \times 10^7 \text{ m}$$ Considering the geometry of the situation, where φ is the maximum latitude that the geostationary satellite can be seen from (people there see it at their horizon since the satellites are in the plane of the equator)  $$\varphi = \cos^{-1}\left(\frac{R_\oplus}{r_{geo}}\right) = \cos^{-1}\left(\frac{6.37 \times 10^6}{4.22 \times 10^7}\right) = 81.3°$$ Hence, only Alert in Canada is at a high enough latitude to have geostationary satellites below their horizon [Note: other geosynchronous satellites could be seen if they were on an orbit inclined to the equator]	1
6.	D It is the Pleiades star cluster in Taurus, with the Messier number M45 (the Orion nebula is M42, next to it is M43 [De Mairan's Nebula], whilst M44 is the Beehive cluster in Cancer)	1
7.	A Aldebaran is the eye of the bull in Taurus	1

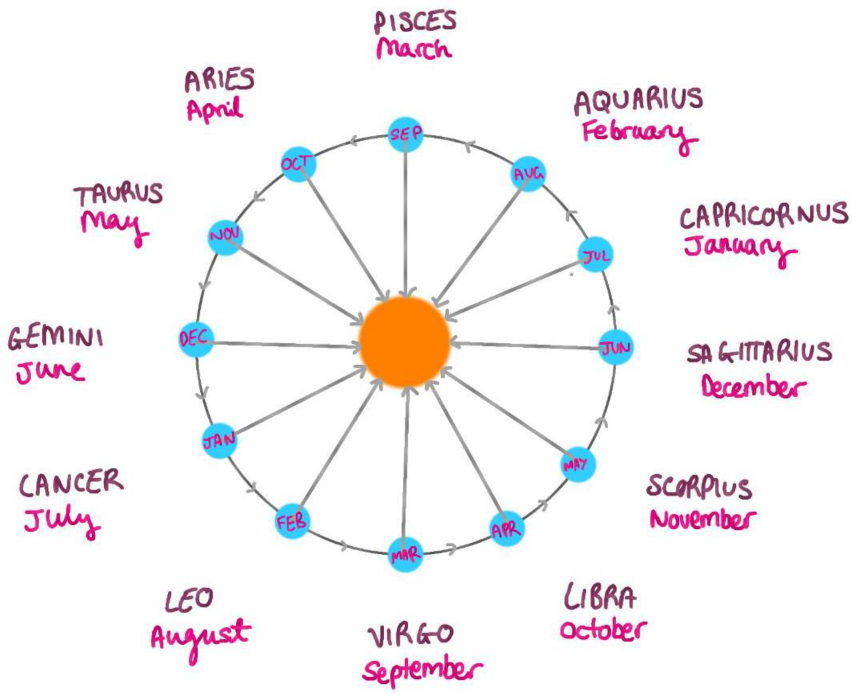
<table>
<tr><td>8.</td><td>D
The nearest (comparable size) galaxy to the Milky Way is the Andromeda Galaxy, and so the constellation needed was Andromeda (the others are Orion, Perseus and Gemini)</td><td>1</td></tr>
<tr><td>9.</td><td>B



Given the Moon is in the third quarter it is 90° behind where the Sun is, so the Sun must be a quarter of a year (= 3 constellations) further on in the zodiac. Remembering the order:

$$\text{Capricornus} \rightarrow \text{Aquarius} \rightarrow \text{Pisces} \rightarrow \text{Aries}$$

Having worked out that the Sun must be in Aries, we need to remember which month that corresponds to:

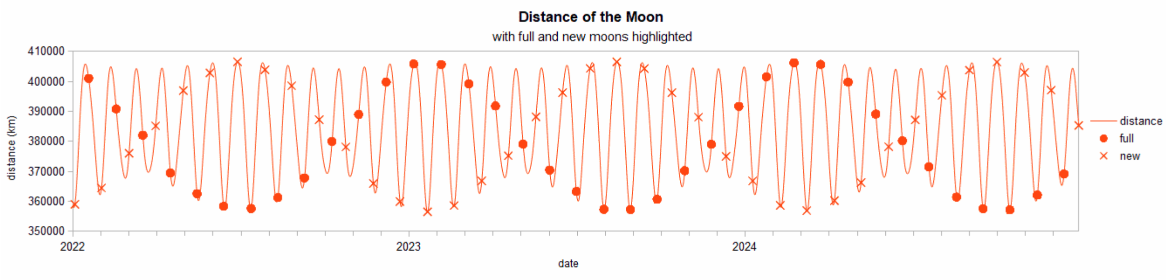


Hence, if the Sun is in Aries then it must be April</td><td>1</td></tr>
</table>

10.	B The adjustment in time between the two locations is determined by the change in longitude $$\frac{\Delta\lambda}{360°} = \frac{\Delta t}{24^h} \therefore \Delta t = \frac{47.51 + 2.59}{360} \times 24 = 3.34 \text{ hours} = 3 \text{ h } 20 \text{ mins}$$ Since Antananarivo is further East of Guernsey then it will be **ahead** The date is 21st June, which is the June solstice, so will be the longest day in Guernsey (northern hemisphere) and the shortest in Antananarivo (southern hemisphere), but their culminations will only be different by Δt. Since it is close to the Greenwich Meridian, culmination time in Guernsey will be approximately 12:00 UT, so culmination time in Antananarivo will be 12:00 – 3h20 = 08:40 UT The sunset time in Guernsey is 21:00 BST = 20:00 UT (since BST = UTC+1), so about 8 hours after culmination. If Antananarivo was on the equator sunset would be about 6 hours after culmination (so 08:40 + 6 = 14:40), and if it was at the same latitude as Guernsey but South (i.e. 49.45° S) then it would be 4 hours after culmination (by symmetry since it's a solstice [6+2 in summer, 6-2 in winter]) and so 08:40 + 4 = 12:40 Since the real latitude of Antananarivo (18.88° S) is between these two cases, it must be earlier than 14:40 but not as early as 12:40, leaving just 14:20 as the only possible answer [Antananarivo is close enough to the equator that its day length doesn't vary much – even though this is its winter solstice it still has about 11 hours of daylight, unlike the 8 hours of daylight in Guernsey on its winter solstice]	1
Section B		**10**
11.	a) The final speed is 1/5th escape velocity $$v_{final} = \frac{1}{5}v_{esc} = \frac{1}{5}\sqrt{\frac{2GM_\oplus}{R_\oplus}} = \frac{1}{5} \times \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{6.37 \times 10^6}}$$ $$= 2240 \text{ m s}^{-1}$$ The centripetal acceleration is $$a = \frac{v_{final}^2}{r} = \frac{2236^2}{45} = 1.11 \times 10^5 \text{ m s}^{-2}$$ Since $g = 9.81 \text{ m s}^{-2}$ $$\therefore a = \boxed{1.13 \times 10^4\, g}$$ [Since the question was a 'show that', the final answer must be given to **at least 2 s.f.** to get the final mark] This is a really high acceleration for the onboard electronics of any satellite to have to cope with and means some fragile structures like large solar panel arrays will need to be stored carefully.	**[3]** 1 1 1

	b) Period of motion (by considering distance travelled in a circle) $$T = \frac{2\pi r}{v_{final}} = \frac{2\pi \times 45}{2236} = 0.126 \text{ s}$$ Hence, the rotational frequency will be $$f = \frac{1}{T} = \frac{1}{0.126} = \boxed{7.91 \text{ revolutions s}^{-1}}$$ [Accept s^{-1} or Hz for the unit]	**[2]** 1 1
12.	a) Difference between 12 lunar months and a year $$365 - (12 \times 29.53) = 10.64 \text{ days}$$ Average time between 365-day periods that contain 13 full moons $$\frac{29.53}{10.64} = 2.78 \text{ years}$$ Since only 25% of blue moons will be supermoons the average time to wait is $$2.78 \times 4 = \boxed{11.1 \text{ years}}$$ [There are quite a wide variety of ways to approach this question – please credit any sensible approaches that get between $\sim 9 - \sim 11$ years. The final mark can be awarded as an ecf for showing they understand how to use the 25% probability of a full moon being a supermoon. The first two marks can be awarded for an approach that shows the probability of a blue moon is $\sim 3\%$ e.g. average month length = $\frac{365}{12} = 30.42$ days, so probability of a blue moon is $\frac{30.42-29.53}{30.42} = 0.029 \approx 3\%$] The actual time between super blue moons is quite irregular, sometimes as long as 20 years. The next real super blue moons will be in a pair – January and March 2037.	**[3]** 1 1 1
	b) The percentage change in angular diameter must be the same as the percentage change between the perigee and apogee, so $$r_a = 1.141 r_p$$ $$\therefore a(1+e) = 1.141a(1-e)$$ $$\therefore 1 + e = 1.141 - 1.141e$$ $$\therefore e = \frac{0.141}{2.141} = \boxed{0.0659}$$ [The first mark is for applying the data to the expressions for apogee and perigee and cancelling the Moon's semi-major axis, whilst the second mark is for a value of the eccentricity]	**[2]** 1 1

<table>
<tr><td></td><td>This eccentricity is a little larger than the normal quoted value of 0.0549 as that is a mean value – the Moon's eccentricity varies over time (mainly due to solar perturbations), and so does its perigee and apogee distance


</td><td></td></tr>
<tr><td>Section C</td><td></td><td>10</td></tr>
<tr><td>13.</td><td>a)

Converting the emitted wavelength 150 nm to an observed wavelength
$$\lambda_{obs} = \lambda_{emit}(1+z) = 150(1+10.6034) = 1740\text{ nm } (= 1.74\ \mu\text{m})$$</td><td>[5]

1</td></tr>
<tr><td></td><td>[This mark can be awarded for correct conversion of 140 nm or 160 nm instead]

Reading off the graph at $\lambda_{obs} = 1.74\ \mu\text{m}$
$$F_\lambda = 1.3 \times 10^{-20}\text{ erg s}^{-1}\text{ cm}^{-2}\text{ Å}^{-1}$$</td><td>1</td></tr>
<tr><td></td><td>[Allow any value in the range 1.2 - 1.4 × 10^{-20} erg s^{-1} cm^{-2} $Å^{-1}$]

Calculating the spectral flux density in Jy
$$F_\nu = 3.34 \times 10^6 \lambda^2 F_\lambda = 3.34 \times 10^6 \times 1740^2 \times 1.3 \times 10^{-20}$$$$= 1.32 \times 10^{-7}\text{ Jy}$$</td><td>1</td></tr>
<tr><td></td><td>Converting this into an apparent magnitude
$$m = -2.5\log\left(\frac{F_\nu}{3631}\right) = -2.5\log\left(\frac{1.32\times10^{-7}}{3631}\right) = 26.1$$</td><td>1</td></tr>
<tr><td></td><td>Finally, the absolute magnitude is then
$$\mathcal{M} = m - 5\log\left(\frac{d_L}{10}\right) + 2.5\log(1+z)$$$$= 26.1 - 5\log\left(\frac{113150\times10^6}{10}\right) + 2.5\log(1+10.6034)$$$$= \boxed{-21.5}$$</td><td>1</td></tr>
<tr><td></td><td>[Since it was a 'show that' question, final answer must have at least 3 s.f.]
Allowed range of values
$$F_\lambda = 1.2\times10^{-20} \rightarrow F_\nu = 1.21\times10^{-7}\text{ Jy} \rightarrow m = 26.2 \rightarrow \mathcal{M} = -21.4$$$$F_\lambda = 1.4\times10^{-20} \rightarrow F_\nu = 1.42\times10^{-7}\text{ Jy} \rightarrow m = 26.0 \rightarrow \mathcal{M} = -21.6$$
This is really quite bright considering the galaxy must be young as it's in the early Universe – the Milky Way by comparison has an absolute magnitude of about -21.</td><td></td></tr>
</table>

	b)	**[4]**
	At a distance of d = 10 pc	
	$$m_A - m_B = \mathcal{M}_A - \mathcal{M}_B = -2.5\log\left(\frac{b_1}{b_0}\right)$$ $$= -2.5\log\left(\frac{\frac{L_1}{4\pi d^2}}{\frac{L_0}{4\pi d^2}}\right) = -2.5\log\left(\frac{L_1}{L_0}\right)$$ $$\therefore \mathcal{M} - \mathcal{M}_\odot = -2.5\log\left(\frac{L}{L_\odot}\right)$$	1
	$$\therefore L = 10^{\left(\frac{\mathcal{M}-\mathcal{M}_\odot}{-2.5}\right)} L_\odot = 10^{\frac{-21.5-4.74}{-2.5}} L_\odot = 3.15 \times 10^{10}\ L_\odot$$ $$= 1.20 \times 10^{37}\ \mathrm{W}$$	1
	We can get the number of ionising photons emitted each second by dividing this power by the energy of each photon (in the emitted frame) $$N_{ion} = \frac{L}{E_{phot}} = \frac{L}{\frac{hc}{\lambda}} = \frac{1.20 \times 10^{37}}{\frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{150 \times 10^{-9}}}$$ $$= 9.09 \times 10^{54}\ \text{photons s}^{-1}$$	1
	Hence the star formation rate is $$SFR = 1.08 \times 10^{-53} N_{ion} = 1.08 \times 10^{-53} \times 9.09 \times 10^{54}$$ $$= \boxed{98.1\ M_\odot\ \text{year}^{-1}}$$	1
	[First mark is for an algebraic relationship between absolute magnitude and luminosity, whilst second mark is for using it to get the luminosity. If third mark is not awarded then it can be given for a correct calculation of the photon energy for λ_{emit} = 150 nm i.e. $hc/\lambda_{emit} = 1.33 \times 10^{-18}$ J] Allowed range of values / ecf assistance $$\mathcal{M} = -21.4 \rightarrow SFR = 90.6\ M_\odot\ \text{year}^{-1}$$ $$\mathcal{M} = -21.6 \rightarrow SFR = 105.7\ M_\odot\ \text{year}^{-1}$$ $$\mathcal{M} = -21 \rightarrow SFR = 61.7\ M_\odot\ \text{year}^{-1}$$	
	This is actually a really high SFR, mostly due to our many simplifying assumptions. To do it properly you would need to take into account the underlying assumption of how many stars are made of each mass (known as the initial mass function). When Bunker et al. (2023) did this they calculated an SFR a factor of 3 smaller, which is less extreme but still high and shows plenty of star formation is taking place.	
	c)	**[1]**
	Assuming a constant SFR $$t = \frac{M}{SFR} = \frac{10^9}{98.1} = \boxed{1.02 \times 10^7\ \text{years}}$$ [Allow ecf with their SFR from part b)]	1
	This is 10.2 million years so is well within the 430 million years age of the Universe, but further emphasises how rapidly this galaxy has formed. [no mark for comment]	

14.	a)	**[3]**
	Calculating the period of the 6:1 MMR $$T_{6:1} = \frac{1}{6} \times 12.4311 \text{ days} = 49.7 \text{ hours}$$	1
	We can use Kepler's 3rd Law to calculate the period of the 1:3 SOR since within a system $T^2 \propto a^3$ $$\frac{T_{1:3}^2}{T_{6:1}^2} = \frac{a_{1:3}^3}{a_{6:1}^3} \therefore T_{1:3} = T_{6:1} \times \left(\frac{a_{1:3}}{a_{6:1}}\right)^{3/2} = 49.7 \times \left(\frac{4200}{4020}\right)^{3/2} = 53.1 \text{ hours}$$	1
	From this we can work out the rotational period $$T_{rot} = \frac{1}{3} T_{1:3} = \frac{1}{3} \times 53.1 = \boxed{17.7 \text{ hours}}$$	1
	b)	**[3]**
	Using Kepler's 3rd Law again with either the 6:1 MMR or the 1:3 SOR (here we use the former, but the latter is equally valid) $$T_{Q2R} = T_{6:1} \times \left(\frac{a_{Q2R}}{a_{6:1}}\right)^{3/2} = 49.7 \times \left(\frac{2520}{4020}\right)^{3/2} = 24.7 \text{ hours}$$	1
	The number of complete cycles is inversely proportional to the period, and the particles in Q2R will complete fewer cycles than Quaoar will complete rotations, so $$\frac{a}{b} = \frac{1/T_{Q2R}}{1/T_{rot}} = \frac{T_{rot}}{T_{Q2R}} = \frac{17.7}{29.7} = 0.717$$	1
	$$\therefore a : b = \boxed{5 : 7}$$	1
	[An answer of a : b = 3 : 4 scores 0.5 marks for the final marking point]	
	c)	**[4]**
	Using Kepler's 3rd Law with any of the three orbits calculated so far (here we will use the 6:1 MMR) $$M_Q = \frac{4\pi^2}{G}\frac{a^3}{T^2} = \frac{4\pi^2}{6.67 \times 10^{-11}} \times \frac{(4020 \times 10^3)^3}{(49.7 \times 60 \times 60)^2} = 1.20 \times 10^{21} \text{ kg}$$	1
	[This mark may be awarded for earlier working if they calculated the mass in part b)]	
	Given that $\alpha = \left(\frac{4\pi}{\gamma}\right)^{1/3}$ and $\rho_Q = \frac{M_Q}{\frac{4}{3}\pi R_Q}$ then putting it into the given equation for the Roche limit $$d = \left(\frac{4\pi}{\gamma}\right)^{1/3} \times R_Q \left(\frac{\frac{M_Q}{\frac{4}{3}\pi R_Q}}{\rho_{ring}}\right)^{1/3} = \left(\frac{3M_Q}{\gamma \rho_{ring}}\right)^{1/3}$$	1

	$\therefore d = \left(\frac{3 \times 1.20 \times 10^{21}}{1.6 \times 400}\right)^{1/3} = \boxed{1780 \text{ km}}$	1
	So $d < a_{Q2R} < a_{Q1R}$ as expected	1
	[The second mark is for developing an expression for the Roche limit that is independent of the radius of Quaoar – allow one kept in terms of α rather than converting to γ. The fourth mark requires some sort of numerical or mathematical comparison to justify the verification] Quaoar is one of only three minor bodies to have ring systems – the other two are Haumea (a dwarf planet) and Chariklo (a Centaur – an asteroid that orbits between Saturn and Uranus). Interestingly, the 1:3 SOR coincides with their rings too, so it might be that this particular resonance is particularly important in forming rings around small bodies. A map of the whole system is shown below: