# Isaac Physics | Linking Concepts 1.

**Gravitational potential and kinetic energy**

Please note: these solutions are not endorsed by Isaac Physics!

If you want to improve your understanding of the underlying physics then **you** have to work through the questions in the Linking Concepts book from Isaac Physics. There are answers in the back of the book for Question 1 and you can check your other answers by entering them at **isaacphysics.org**. This is just an example of how to work through a chapter.

Objects rising and falling exchange stores of gravitational potential and kinetic energy.

Example context: We can calculate the speed of objects after they have fallen. We can also work out the height to which a projected object rises. The analysis is particularly useful when balls bounce.

| Quantities: | | | | |
|---|---|---|---|---|
| | $h_0$ | starting height (m) | $v_0$ | starting speed (m s$^{-1}$) |
| | $h_1$ | final height (m) | $v_1$ | final speed (m s$^{-1}$) |
| | $m$ | mass (kg) | $g$ | gravitational field strength (N kg$^{-1}$) |
| | $E_K$ | kinetic energy (J) | $E_{GP}$ | gravitational potential energy (J) |
| | $\eta$ | efficiency (no unit) | $E_T$ | total energy (J) |

Equations: $E_K = ½\, m\, v^2$ $\quad E_{GP} = m\, g\, h$ $\quad E_T = E_K + E_{GP}$ $\quad E_{T,after} = \eta\, E_{T,before}$

1.1 In the absence of air resistance, use the above equations to derive expressions for:

a. the speed $v_1$ at the ground if an object was dropped from $h_0$.

$$\cancel{m} g h_0 = \frac{1}{2}\cancel{m} v_1^2 \qquad g h_0 = \frac{v_1^2}{2} \qquad v_1 = \sqrt{2 g h_0}$$

b. the speed $v_1$ at a height $h_1$ if an object had speed $v_0$ at $h_0$.

$$\cancel{m} g h_0 + \frac{1}{2}\cancel{m} v_0^2 = \cancel{m} g h_1 + \frac{1}{2}\cancel{m} v_1^2$$

$$2 g h_0 + v_0^2 = 2 g h_1 + v_1^2$$

$$v_1^2 = v_0^2 + 2 g h_0 - 2 g h_1 \qquad v_1 = \sqrt{v_0^2 + 2g(h_0 - h_1)}$$

c. the greatest height $h_1$ for an object projected up from the ground with speed $v_0$.

$$\frac{1}{2}\cancel{m} v_0^2 = \cancel{m} g h_1 \qquad \frac{v_0^2}{2} = g h_1 \qquad h_1 = \frac{v_0^2}{2g}$$

d. the greatest height $h_1$ for an object projected up from a height $h_0$ with speed $v_0$.

$$\cancel{m} g h_0 + \frac{1}{2}\cancel{m} v_0^2 = \cancel{m} g h_1$$

$$g h_1 = g h_0 + \frac{v_0^2}{2} \qquad h_1 = h_0 + \frac{v_0^2}{2g}$$

e. the greatest height $h_1$ above a hard surface reached by an object dropped from a height $h_0$ if the efficiency of the bounce is $\eta$.

$$\eta \cancel{m} \cancel{g} h_0 = \cancel{m} \cancel{g} h_1$$

$$\eta h_0 = h_1$$

$$h_1 = \eta h_0$$

f. the speed $v_1$ just after a bounce from a hard surface if the speed just before was $v_0$ (if the efficiency of the bounce is $\eta$).

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$$\eta \cancel{\tfrac{1}{2}} \cancel{m} v_0^2 = \cancel{\tfrac{1}{2}} \cancel{m} v_1^2$$

$$\eta v_0^2 = v_1^2$$

$$v_1 = \sqrt{\eta} . v_0$$

1.2 An 800 kg pumpkin falls from 3.4 m. Calculate its speed just before striking the ground.

From 1.1a $\quad v_1 = \sqrt{2 g h_0}$

$$v_1 = \sqrt{2 \times 9.81 \times 3.4}$$

$$v_1 = 8.167$$

$$v_1 = \underline{8.2} \ m s^{-1}$$

1.3 A 60 g tennis ball is hit upwards at 27 m s$^{-1}$. How high will it rise?

From 1.1 c

$$h_1 = \frac{v_0^2}{2g} = \frac{27^2}{2 \times 9.81} = 37.16 = \underline{37}\ \text{m}$$

1.4 A 60 g tennis ball is hit upwards at 27 m s$^{-1}$ from a 25 m rooftop. How fast will it be travelling when it passes the rooftop on the way down?

From 1.1 b

$$v_1 = \sqrt{v_0^2 + 2g(h_0 - h_1)} = \sqrt{27^2 + (2 \times 9.81 \times (25 - 25))}$$

$$v_1 = \sqrt{27^2 + 0}$$

$$v_1 = \underline{27}\ \text{m s}^{-1}$$

1.5 A 3.1 kg brick falls from scaffolding on a building site. A worker 3.5 m above the ground sees it fall past at 6.5 m s$^{-1}$. What is its kinetic energy just before striking the ground?

$$E_{K\,after} = E_{K\,before} + E_{GP\,before}$$

$$E_{K\,after} = \frac{1}{2} m v_0^2 + m g h_0$$

$$E_{K\,after} = \left(\frac{1}{2} \times 3.1 \times 6.5^2\right) + (3.1 \times 9.81 \times 3.5)$$

$$E_{K\,after} = 171.926 \approx \underline{170}\ \text{J}\ (2\ \text{sf})$$

1.6 At what speed will a 4.2 kg lump of clay hit a potter's wheel if it is thrown downwards at 1.1 m s$^{-1}$ from a height 40 cm above the wheel?

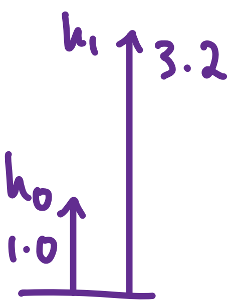
From 1.1 b

$$v_1 = \sqrt{v_0^2 + 2g(h_0 - h_1)} = \sqrt{1.1^2 + (2\times 9.81\times(0.40))}$$

$$v_1 = 3.010 = 3.0 \text{ ms}^{-1}$$

1.7 A worker at ground level throws a 2.2 kg drinks bottle upwards to a thirsty colleague 3.2 m above the ground. It just reaches him, but he fails to catch it, and it falls into an excavated trench 1.6 m below ground level.

a. At what speed did the worker need to throw the bottle if she threw it from the waist, 1.0 m above the ground?

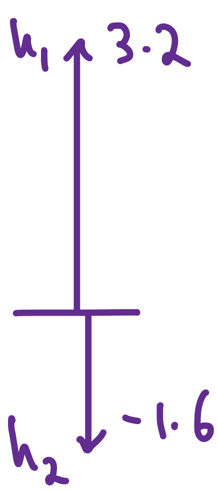



$$\cancel{m}gh_0 + \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}gh_1$$

$$v_0 = \sqrt{2g(h_1 - h_0)}$$

$$v_0 = \sqrt{2\times 9.81\times(3.2-1.0)} = 6.570 = 6.6 \text{ ms}^{-1}$$

b. How fast was it moving when it struck the base of the trench?




$$mgh_1 = mgh_2 + \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

$$v_2 = \sqrt{2\times 9.81\times(3.2-(-)1.6)}$$

$$v_2 = 9.704 = 9.7 \text{ ms}^{-1}$$

1.8 A 5.2 g ball is dropped from 90 cm onto a surface and bounces to a maximum height of 41 cm. Calculate the efficiency, $\eta$.

From 1.1e

$$h_1 = \eta h_0 \qquad \eta = \frac{h_1}{h_0} = \frac{41}{90} = 0.4556 = \underline{0.46}$$

1.9 How fast would the ball, in question 1.8 above, rebound if it struck the surface at 2.5 m s⁻¹?

From 1.1f

$$v_1 = \sqrt{\eta}.v_0 = \sqrt{0.4556} \times 2.5 = 1.687 = \underline{1.7}\ m s^{-1}$$

1.10 How high would a ball bounce if it struck an $\eta$ = 0.75 surface at 13 m s⁻¹?

$$\eta \frac{1}{2} \cancel{m} v_0^2 = \cancel{m} g h_1$$

$$h_1 = \frac{\eta v_0^2}{2g} = \frac{0.75 \times 13^2}{2 \times 9.81} = 6.460 = \underline{6.5}\ m$$