A Level Physics Online

OCR B Physics – H557

Module 5: Rise and Fall of the Clockwork Universe

You should be able to demonstrate and show your understanding of:	Progress and understanding: 1	2	3	4
5.1 Models and Rules				
5.1.1 Gravitation and Circular Motion				
One radian is the angle subtended at the centre of a circle by an arc of the circle where the length of the arc equals the radius of the circle				
Converting from degrees to radians, $\times \frac{2\pi}{360}$				
Converting from radians to degrees, $\div \frac{2\pi}{360}$				
Arc length = radius x θ (l = rθ)				
When θ is very small, sinθ ≈ tanθ ≈ θ				
Consider a particle moving along a circular path. The velocity is changing as the particle is changing direction. Even though the speed is constant, there is still an acceleration as a force is still applied to the particle. This force is centripetal which produces a centripetal acceleration. Centripetal means acting towards the centre of the orbit. $F = \frac{mv^2}{r} \quad and \quad a = \frac{v^2}{r}$ Note: A centripetal force is NOT a new force, it is the component of the net force directed towards the centre of the orbit. It could be due to a reaction force, spring force, tension etc				
Angular Displacement, Δθ: The angle moved through relative to a specific axis				
Angular Velocity, ω: The rate of change of angular displacement with respect to time $\omega = \frac{\Delta\theta}{\Delta t} \qquad \omega = 2\pi f = \frac{2\pi}{T}$				

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We can derive an equation to relate linear and angular velocity. If a particle is moving in a circular orbit with linear (tangential) velocity, v, it moves a distance l=rθ (from above) in a time, t; $$v = \frac{dl}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega \qquad so, \qquad v = \omega r$$ Note: r is taken outside the differential because it is a constant multiplier				
Using this new equation, we can substitute it into the above equations for centripetal force and acceleration to get; $$F = mr\omega^2 \qquad a = \omega^2 r$$				
The gravitational force is given by the equation below, where G is the gravitational constant and G = 6.67 x 10^{-11} Nm^2kg. There is a negative sign because the force due to gravity is always attractive $$F_{grav} = -\frac{GMm}{r^2}$$ Where M and m are the masses concerned in orbit and r is the distance between the mass concerned and the centre of orbit				
Test mass: A mass small enough so that it does not affect the surrounding gravitational field				
If an object is put into a gravitational field say <u>it is subject to a force</u>, NOT it feels a force				
Gravitational Field Strength, g (Units: Nkg^{-1}): The magnitude and direction of the force on 1kg at a given point in a gravitational field. Field lines represent the direction and magnitude of it on a diagram; $$g = -\frac{GM}{r^2}$$ Note: g varies with an inverse square law as shown by the equation				
Gravitational Field inside the Earth: Proportional to the distance from the centre. The mass of Earth above a point inside the planet does not have a gravitational effect. Considering a point, P, inside the Earth; $$g_P = -\frac{GM_p}{r_P^2}$$ $$where\ M_P = \rho\frac{4}{3}\pi r_P^3\ is\ the\ mass\ of\ Earth\ below\ P, hence$$ $$g_P = -\rho\frac{4}{3}G\pi r_P$$				
Uniform Field: A field where there is the same magnitude and direction of field everywhere. For example, near the Earth's surface is a uniform field				

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Potential in a Uniform Field: For a falling particle; $$\Delta E_P = \Delta E_k \quad \rightarrow \quad \Delta mgh = \Delta \frac{1}{2}mv^2 \quad \rightarrow \quad \Delta gh = \Delta \frac{1}{2}v^2$$				
Geostationary Satellites: Stay in the same place in the sky. Orbit the Earth above the equator once a day. An orbit that is too low will result in it moving too fast, an orbit that is too high will result in it moving too slow. A stable orbit will be achieved if the velocity of the satellite equals the rate of Earth's rotation, this occurs if the centripetal force equals the gravitational force; $$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad then\ substitute\ in\ v = \frac{2\pi r}{T};$$ $$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \rightarrow r^3 = \frac{GMT^2}{4\pi^2} \quad this\ results\ in;$$ $$r^3 \propto T^2$$ This is Kepler's third law				
The orbital period of the geostationary satellite can also be determined; $$T = \frac{2\pi r}{v} \quad \rightarrow \quad T^2 = \frac{4\pi^2 r^2}{v^2} \quad then\ substitute\ in\ v^2 = \frac{GM}{r}$$ $$T^2 = \frac{4\pi^2 r^3}{GM} \quad then\ substitute\ in\ g = \frac{GM}{r^2}$$ $$T = 2\pi\sqrt{\frac{r}{g}}$$				
Gravitational Potential, V (Units: Jkg^{-1}): The gravitational energy per kg of material. For a uniform near Earth, $\Delta V = 9.81 Jkg^{-1}$ for every metre raised $$V = -\frac{GM}{r}$$				
Equipotential Surfaces: A continuous surface of the same gravitational potential. In a uniform field, the spacing between equipotentials is equal. Along an equipotential, not gravitational force acts because the potential energy is not changing hence no work is done. The direction of the gravitational field is always perpendicular to the equipotentials. The change in potential between two points (on different equipotentials) is the same irrespective of the route taken. This is true for uniform and non-uniform fields				
Gravitational Potential Wells: For calculations concerning celestial objects, zero is set as the potential energy per kg at infinity. Gravitational potential increases the further you get from Earth. A more massive object has a more negative potential at its surface. For a spacecraft travelling from a point of net gravitational neutrality to Earth, its kinetic energy increases and its potential energy decreases as it 'falls down' the potential well. A point of net gravitational neutrality is where there is zero net force acting				

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Escape Velocity: For a particle to escape a gravitational potential well, $E_{TOT} = E_P + E_k \geq 0 \quad \rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{r^2}$ Rearrange for v gives the minimum escape velocity, $$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2}v_{orb}$$ To aid in escaping a potential well there are ways of gaining energy; the Earth is rotating so the launch site has kinetic energy already. A gravity assist/slingshot is where a body gains kinetic energy through interacting with another planet				
Orbital Velocity: If a body has a higher velocity than the orbital velocity, it will move to a higher orbit. If the orbital velocity is greater than the escape velocity for the radius of orbit, the body will leave the planet's orbit $$v_{orb} = \sqrt{\frac{GM}{r}}$$				
Radial Field Equations: Some of these have already been covered above, ut just to make explicit that these apply to radial fields (where the field lines meet at a central point), they are copied here; $$F_{grav} = -\frac{GMm}{r^2} \quad g = -\frac{GM}{r^2} \quad E_P = -\frac{GMm}{r} \quad V = -\frac{GM}{r}$$ The only one not covered previously is the equation for gravitational potential energy. A simple way to think about it is to use the work equation, W=Fr, where F is the gravitational force, so an r on the bottom cancels with the one on the top. For a radial field, the spacing of equipotentials separated by equal potential, V, increases with distance from a point mass. Greater spacing means a weaker field				
Potential Gradient: If a mass moves a distance Δr (not along an equipotential), its potential changes by ΔV. The force required to move the mass vertically upwards at a constant velocity is; $$F = -mg \text{ and the work done is}, W = Fd = \Delta E_P = Vm \text{ so};$$ $$-mg\Delta r = \Delta Vm;$$ $$g = -\frac{\Delta V}{\Delta r} \text{ and as } \Delta r \rightarrow 0;$$ $$g = \frac{dV}{dr}$$ This is the potential gradient (for a radial field). The radial component of the field at any point in space equals the <u>negative</u> of the potential gradient at that point				
A g-r graph is in the fourth quadrant as g is always negative and r is positive. The area of the graph between the graph and the x-axis gives ΔV				

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An F-r graph is also in the fourth quadrant, for similar reasons. The area of the graph between the graph and the x-axis gives ΔE_P				
A V-r graph is in the fourth quadrant, for similar reasons. The gradient of the graph gives -g (The gradient is positive so to find g it is the negative of the gradient)				
An E_P-r graph is in the fourth quadrant, for similar reasons. The gradient of the graph gives -F (The gradient is positive so to find F it is the negative of the gradient)				