A Level Physics Online

OCR B Physics – H557

Module 5: Rise and Fall of the Clockwork Universe

You should be able to demonstrate and show your understanding of:	Progress and understanding: 1	2	3	4
5.1 Models and Rules				
5.1.1 Radioactivity and Capacitors				
Radioactivity is where unstable nuclei breakdown via radioactive decay, emitting ionising radiation in the process (alpha: helium nucleus, beta: fast moving electrons, gamma: high energy photons)				
Activity, A (Bq or s^{-1}): The number of nuclei decaying per second OR the rate at which nuclei decay OR the count rate per second on a Geiger counter $$A = \frac{dN}{dt} = \frac{\Delta N}{\Delta t}$$				
Count rate is usually smaller than activity because not emitted particles will be detected. However, activity and count rate are proportional, if activity halves then count rate will halve too On a 'half life graph' (N, number of remaining nuclei on the y axis and t along the x axis) the activity is found by drawing a tangent to the curve at a given time, t				
Half-life: The time taken for the number of nuclei in a sample to fall to half the original value OR the time it takes for the activity to halve OR the average time for half of the sample, from any starting point, to decay				
For one $t_{1/2}$, N reduces by a factor 2. In L half-lives, the number of nuclei remaining reduces by a factor of 2^L				
Radioactive decay is a spontaneous and random event that cannot be predicted. The rate of decay is not affected by external factors. An experiment to show the nature of decay can be done involving dice with one side painted a different colour. We can't predict which dice will land on the coloured side (i.e. decay). However, the outcome of many dice is predictable, after all the dice are thrown, roughly 1/6 should show the coloured side (have decayed) so this shows decay is both predictable and random				
On a Geiger-Muller tube, one count represents one ionisation event within the tube / one decay within the tube				

You should be able to demonstrate and show your understanding of:	Progress and understanding:			
	1	2	3	4
Decay Constant, λ (Units: s^{-1}): The probability that a nucleus will decay during a time interval of one second $$A = \lambda N$$ For a 'half life graph' (N against t), a steeper gradient means a greater λ				
By equating the two equations for activity, we get; $\frac{\Delta N}{\Delta t} = -\lambda N \; (equation\ 1)$ → $\Delta N = -\lambda N \Delta t$ (equation 1.1) This gives an equation that tells us the number of nuclei that decay, ΔN, in a small time interval, Δt. The negative sign shows there is a negative correlation/gradient on the 'half-life graph'. From the equation on the left we can say that activity is proportional to the number, N, of nuclei remaining				
Exponential Change: Rate of change of a quantity is proportional to the value of that quantity				
An iterative model can be constructed to model radioactive decay using equation 1.1 above. Let us consider an example where N_0 = 1000 and λ = 0.6s^{-1} and we want to find the number of nuclei remaining, N after 3 seconds 1)The number decayed in the 1st second is ΔN=1000x0.6= 600, so N_{new} = 400 2)The number decayed in the 2nd second is ΔN=400x0.6=240, so N_{new} = 160 3)The number decayed in the 3rd second is ΔN=160x0.6=96, so N_{new} = 64 So after 3 seconds, 64 undecayed nuclei remain in the sample. -When an iterative calculation is plotted on a graph, the line between each successive pair of points is linear, showing that the model assumes the activity is constant during each interval Δt. This isn't true in a real example, so to improve the model Δt is made as small as possible				
It is difficult to accurately predict the number of nuclei that will decay in a sample with few nuclei and a long half life because the random nature of decay becomes more apparent. The uncertainty in the number of nuclei that will have decayed becomes very large (if there were few dice in the example above we could not accurately say that approximately 1/6 of the dice will have 'decayed'). There are not enough nuclei to reduce the random error				
$$\lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \frac{dN}{dt}$$ In words, as Δt becomes smaller and smaller (tends to 0), the change in N over the change in time becomes the derivative of N with respect to time. This is just a piece of mathematical notation, not strictly in the course but I thought it useful to put in (covered in C1 Edexcel A-level Mathematics under 'Differentiation from First Principles')				
To exponential decay equation relating number of nuclei remaining and the original number of nuclei is; $N = N_0 e^{-\lambda t}$ which is analogous to $A = A_0 e^{-\lambda t}\ and\ R = R_0 e^{-\lambda t}$ where A is the activity and R is the count rate (on a Geiger Counter/GM Tube) because N is proportional to A and A is proportional to R				

You should be able to demonstrate and show your understanding of:	Progress and understanding:			
	1	2	3	4
For any equation with an exponential term to work, the values involved in the exponent (the power part of the e term) must have units that cancel. So in this case, the units of λ and t cancel				
We can form an equation relating half-life and the decay constant, which is useful because we cannot measure λ directly, but we can measure $t_{1/2}$, so it gives another, simpler way of calculating λ; 1) $A = A_0 e^{-\lambda t}$ 2) $lnA = ln\ (A_0 e^{-\lambda t}\ )$ 3) $lnA = \ln(A_0) + \ln\left(e^{-\lambda t}\right)$ 4) $lnA = \ \ln(A_0) - \lambda t$ 5) $ln\left(\frac{A}{A_0}\right) = -\lambda t$ 6) when t = $t_{1/2}$, A = $A_0/2$, therefore $\ln\left(\frac{(\frac{A_0}{2})}{A_0}\right) = -\lambda t_{1/2}$ 7) $ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$ 8) $\ln(1) - \ln(2) = -\lambda t_{1/2}$ 9) $-\ln(2) = -\lambda t_{1/2}$ (because ln(1) = 0) 10) $t_{1/2} = \frac{\ln(2)}{\lambda}$				
A graph of ln(A) against t is plotted for when there is a large half-life because we cannot wait for the activity to halve if a sample has a half-life of thousands of years; so ΔA is measured over a short Δt. The decay constant can be found by taking the positive value of the gradient. The graph has a linear equation (of the form y =mx + c) $lnA = -\lambda t + \ln(A_0)$				
Exponential Decay Tests: -Constant percentage change -Constant half life -Take the ln of the e^x graph, and a straight line should be produced				
Capacitor: An electrical component used to keep charge separated. It is comprised of a pair of electrical conductors separated by an insulator (a dielectric). Charge does not flow through a capacitor because each conductor is separated by the insulator.				
Charging a Capacitor: 1) When the cell is switched on, charges are given energy to flow around the circuit, i.e. a potential difference is created in the circuit 2) As negative charges come in contact with the first plate of the capacitor, the plate is made increasingly negative (causing the opposite plate to become positive) 3) As more charge flows, the current tends to zero amps, and the voltage on the capacitor tends to the voltage on the cell				

You should be able to demonstrate and show your understanding of:	**Progress and understanding:**			
	1	**2**	**3**	**4**
4) This is because like charges repel, and the more negative charges there are on the first plate of the capacitor, the harder it is for charge to flow from the battery, until charge won't flow anymore (as V across the capacitor increases, current in the circuit decreases) 5) At this point, the voltage across the capacitor is a maximum – we assume this to be the voltage supplied by the cell ($V_{capacitor}$ = ε of supply)				
Capacitors can be used instead of batteries in some cases to provide a high, instantaneous supply of current e.g. during power cuts				
Charge on the plates of a capacitor is proportional to the voltage across the capacitor, so a Q-V graph for a capacitor will be linear. On a Q-V graph, the capacitance is the gradient				
Capacitance, C (Units: F, Farads): The ratio of the change in charge stored in an electrical system to the corresponding change in the potential difference OR the ability of a system to store an electric charge $$C = \frac{Q}{V}$$ [Note: Capacitance is usually given in pF or µF so recall that pico is 10^{-12} and femto is 10^{-15}, just remember that femto and fifteen both start with an f] -A capacitor cannot be described as fully charged because a greater p.d. across the capacitor will result in a greater charge stored on the capacitor, they are proportional				
When a capacitor is set to discharge, the charge on the negative plate to the positive plate through the components in a circuit e.g. a bulb will light up, showing energy has been transferred from the capacitor to the bulb				
The energy transferred when a charge, Q, moves through a voltage, V, in free space is E = QV (from the definition of voltage). However, the energy stored on a capacitor is E = 0.5QV The more voltage you apply across the capacitor, the more charge there is stored on the plate. The electrons cause resistance (due to electrostatic repulsion) so an increasing amount of work must be done on a charge to get it onto the capacitor, as V increases, Q increases $$E_{stored} = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{V}$$ These variations are useful because Q is difficult to measure, C is from the manufacturer and V is easy to measure				
The energy stored on a capacitor is equal to the area under a Q-V graph. This can be found either by using the area of a triangle, A_T=0.5bh, or using the trapezium rule				
P = IV <u>cannot</u> be used when finding the power for a capacitor, P = E/t is used instead for the average power over a time interval, t				
When the maximum p.d. across the capacitor is exceeded, the insulating medium could break down, allowing current to flow through the capacitor.				

You should be able to demonstrate and show your understanding of:	Progress and understanding:			
	1	2	3	4
For a small time interval, Δt, and a steady current we can find a differential equation relating the current to the capacitance 1) $\Delta Q = -I\Delta t$ (rearrangement of definition of current, negative shows we are discharging the capacitor) 2) $\Delta Q = -\frac{V}{R}\Delta t$ (substituting Ohm's Law) 3) $\Delta Q = -\frac{Q}{RC}\Delta t$ (substituting V = Q/C) 4) $\frac{\Delta Q}{\Delta t} = -\frac{Q}{RC} \; (equation\ 2)$ [Note: This only applies for discharging a capacitor] [Note: This equation shows an exponential relationship as R and C are constant so rate of change of Q is proportional to Q]				
An iterative model can be constructed to model the discharging of a capacitor using equation 2 above. Let us assume we are given the values/have measured the values of C, R, Q_0, V_0, and Δt; 1) With a charge, Q_0, on the capacitor, $V_{resistor} = Q_0/C$ 2) Find the current, I, flowing in the circuit using $I_0 = V_0/R$ 3) Find the charge, Q, leaving the capacitor in Δt using ΔQ = IΔt 4) $Q_{new} = Q_0 - \Delta Q$ 5) Start back at 1 with Q_0 being replaced by Q_{new} [Note: Q_{new} is the charge stored on the capacitor during discharge after a time interval Δt] -When an iterative calculation is plotted on a graph, the line between each successive pair of points is linear, showing that the model assumes the current is constant during each interval Δt. This isn't true in a real example, so to improve the model Δt is made as small as possible				
$\lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = -\frac{Q}{RC}$ In words, as Δt becomes smaller and smaller (tends to 0), the change in Q over the change in time becomes the derivative of Q with respect to time. As above, this is just a piece of mathematical notation, not strictly in the course but I thought it useful to put in (covered in C1 Edexcel A-level Mathematics under 'Differentiation from First Principles')				
When solved, this differential equation gives the result that $Q = Q_0 e^{-\frac{t}{RC}}$ -Q can be replaced with V or I (and hence Q_0 with V_0 or I_0) because Q is proportional to V and V is proportional to I				
After 'RC' seconds, the charge on the capacitor has been reduced to 1/e of its original value; $Q = Q_0 e^{-1}$ hence $\frac{Q}{Q_0} = e^{-1} = \frac{1}{e}$				

You should be able to demonstrate and show your understanding of:	**Progress and understanding:**			
	1	**2**	**3**	**4**
Time Constant, τ, (Units: seconds): The time taken for the p.d. across the capacitor and the charge stored on the capacitor to fall to 1/e of their original value when discharging through a resistance, R. $$\tau = RC$$ -After nτ seconds, the fraction of the original charge remaining on the capacitor, $Q/Q_0 = 1/e^n$				
We can form an equation relating time and the time constant by finding how long it takes for a capacitor to lose half of its charge; 1)$Q = Q_0 e^{-\frac{t}{RC}}$ 2) $\frac{Q_0}{2} = Q_0 e^{-\frac{t}{RC}}$ 3) $\frac{1}{2} = e^{-\frac{t}{RC}}$ 4) $\ln\left(\frac{1}{2}\right) = \ln\left(e^{-\frac{t}{RC}}\right)$ 5) $\ln(2) = \frac{t}{RC}$ 6) t = ln(2)RC = ln(2)τ				
When charging a capacitor, the p.d. across the capacitor rises and the p.d. across the resistor falls. As the p.d. across the resistor falls, so does the current in the circuit, so the rate of charging of the capacitor also falls				
For a series circuit with a cell of $V = V_0$, a capacitor of $V = V_C$, and a resistor of $V = V_R$, an 'exponential equation' for charging a capacitor can be formed 1) $V_C = V_0 - V_R$ 2) Using $I = I_0 e^{-\frac{t}{RC}}$ and the fact that I_R is proportional to V_R (by Ohm's Law), we can write that $V_R = V_0 e^{-\frac{t}{RC}}$. We can say V_R here because when a capacitor is charging, V_R is decreasing exponentially so we can use the decay form of the equation, whereas V_C is increasing as below 3) Using the result from step 1) this can be rewritten as $V_C = V_0 - V_0 e^{-\frac{t}{RC}}$ $$V_C = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$ 4) Because V_C is proportional to Q_C (the charge stored on the capacitor) we can write $$Q_C = Q_0\left(1 - e^{-\frac{t}{RC}}\right)$$ where $Q_0 = CV_0$				
Charging a capacitor is not an example of exponential growth because the charge is increasing at a decreasing rate, not increasing at an increasing rate. Although the equation has an exponential term in it, charging a capacitor is not 'exponential' by the definition of the word				