British Astronomy and Astrophysics Olympiad 2023-24

Astro Challenge Paper

November / December 2023

Instructions

Time: 1 hour (30 marks).

Questions: Answer all questions in Sections A and B, but only **one** question in Section C.

Marks: Marks allocated for each question are shown in brackets on the right. Working must be shown in order to get full credit, and it is helpful to write down numerical values of any intermediate steps.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Eligibility: All sixth form students (or younger) are eligible to sit any BAAO paper.

Further Information about the British Astronomy and Astrophysics Olympiad

To progress to the next stage of the BAAO, you must take the BPhO Round 1 (which is a general physics problem paper) in November 2023 and / or the BAAO Round 1 in January 2024. Those achieving a Top Gold in either paper will be invited to take the BAAO Round 2 on **Tuesday 20th February 2024**. *A Distinction in this paper, whilst highly commendable, no longer grants access to BAAO Round 2.*

To be eligible for a grade in this paper, it should be sat under exam conditions and all papers should have their results recorded using the online form by **Friday 15th December 2023**.

You are recommended to show your working, and to look up at a clear night sky a few days beforehand.

This paper has more than an hour's worth of questions. You are encouraged to have a go at as many as you can and to follow up on those that you do not complete in the time allocated.

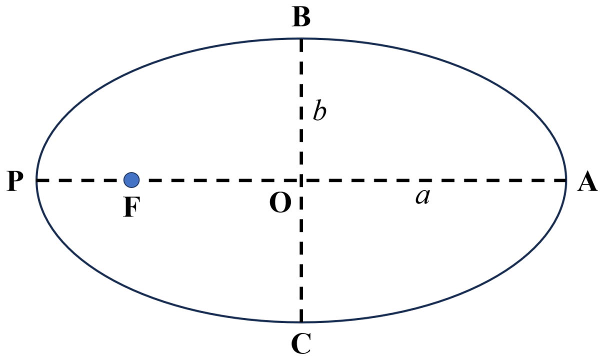
The BAAO are very proud to be sponsored by G-Research

Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_\odot$	$6.96 \times 10^8\,\mathrm{m}$
Radius of the Earth	$R_\oplus$	$6.37 \times 10^6\,\mathrm{m}$
Mass of the Sun	$M_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_\oplus$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_\odot$	$3.83 \times 10^{26}\,\mathrm{W}$
Absolute magnitude of the Sun	$\mathcal{M}_\odot$	4.74
Hubble constant	H_0	70 km s^{-1} Mpc^{-1}
Gravitational constant	G	6.67×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k_B	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_p	1.67×10^{-27} kg
Electron rest mass	m_e	9.11×10^{-31} kg
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)
πab	area of the ellipse

Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

Brightness (Intensity):

$$b = \frac{L}{4\pi d^2}$$

Magnitudes:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - \mathcal{M} = 5\log\left(\frac{d}{10}\right)$$

Distance-Parallax Relation:

$$d = \frac{1}{p}$$

Redshift:

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

Hubble's Law:

$$v = H_0 d$$

Section A: Multiple Choice

Write the correct answer to each question. Each question is worth 1 mark. There is only one correct answer to each question. **Total: 10 marks**.

1. SaxaVord is due to be one of the UK's newest spaceports, with first launches planned in late 2023. Where is it based?

   A. Cornwall, England
   B. Llanbedr, Wales
   C. Sutherland, Scotland
   D. Unst, Shetland Islands

2. In a given system, for all circular orbits of period T and orbital speed v, which of the following quantities is the same for each orbit?

   A. Tv
   B. $Tv^{3/2}$
   C. Tv^2
   D. Tv^3

3. The recent supernova SN 2023ixf was one of the closest to Earth of the past decade, reaching an apparent magnitude of 10.8 despite being in the galaxy M101, about 21 million light years away. What was the absolute magnitude of this supernova?

Figure 1: Credit: Craig Stocks.

   A. -18.2
   B. -19.5
   C. -20.8
   D. -101

4. In August 2023, India became the fourth country to successfully land a spacecraft on the Moon. The Chandrayaan 3 mission also involved the Pragyan rover, which was fully dependent on its small solar panel to provide it with the 50 W necessary to move around the lunar surface and do experiments. Assuming it was pointing at the Sun the whole time, what was the minimum solar panel area needed?

   A. 29 cm^2
   B. 370 cm^2
   C. 1160 cm^2
   D. 4640 cm^2

5. From which of these settlements can a geostationary satellite **not** be seen?

   A. Longyearbyen, Svalbard, Norway (latitude $= 78.20°$ N)
   B. Nagurskoye, Franz Josef Land, Russia (latitude $= 80.80°$ N)
   C. Alert, Ellesmere Island, Canada (latitude $= 82.47°$ N)
   D. Geostationary satellites are visible from all of these latitudes

6. What is the Messier number of this object?

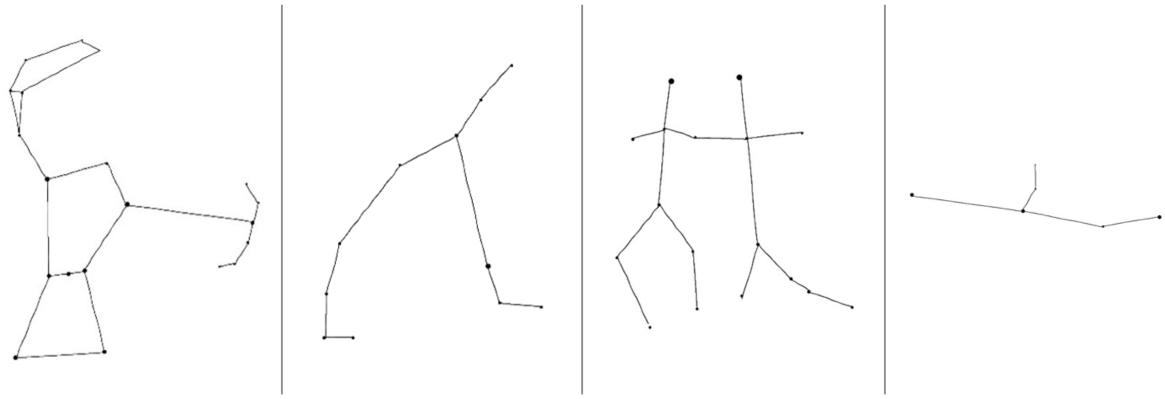
   A. M42
   B. M43
   C. M44
   D. M45

7. Which of the following stars is in the constellation Taurus?

   A. Aldebaran
   B. Castor
   C. Rigel
   D. Sirius

8. Which of these constellations contains the nearest (comparable size) galaxy to the Milky Way.

A. B. C. D.



9. An observer in the UK sees a third quarter moon in the constellation Capricornus. What is the month?

   A. January
   B. April
   C. July
   D. October

10. On the 21st June the Sun sets in Guernsey (49.45° N, 2.59° W) at 21:00 BST (British Summer Time, BST = UT+1). At what time UT (approximately) did the Sun set as observed from Antananarivo, Madagascar (18.88° S, 47.51° E) on the same day?

    A. 12:40
    B. 14:20
    C. 14:40
    D. 16:40

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Section B: Short Answer

Each short question is worth 5 marks. **Total: 10 marks**.

SpinLaunch

11. SpinLaunch is an engineering project that aims to build a new type of rocket launching system that will allow satellites up to 200 kg to be put into orbit far more cost effectively than with current chemical fuel rockets. The payload is accelerated in a 45 m radius circle, using an electric motor system in a vacuum chamber, until it reaches about 1/5th escape velocity and is released, firing its onboard rocket once it reaches 60 km altitude to accelerate it further and ending up in orbit.

    Although only sub-orbital test flights have successfully taken place so far from the Spaceport America site in New Mexico (with a chamber roughly a third of the size), planning for the full scale orbital launch facility is well underway (see Figure 2).

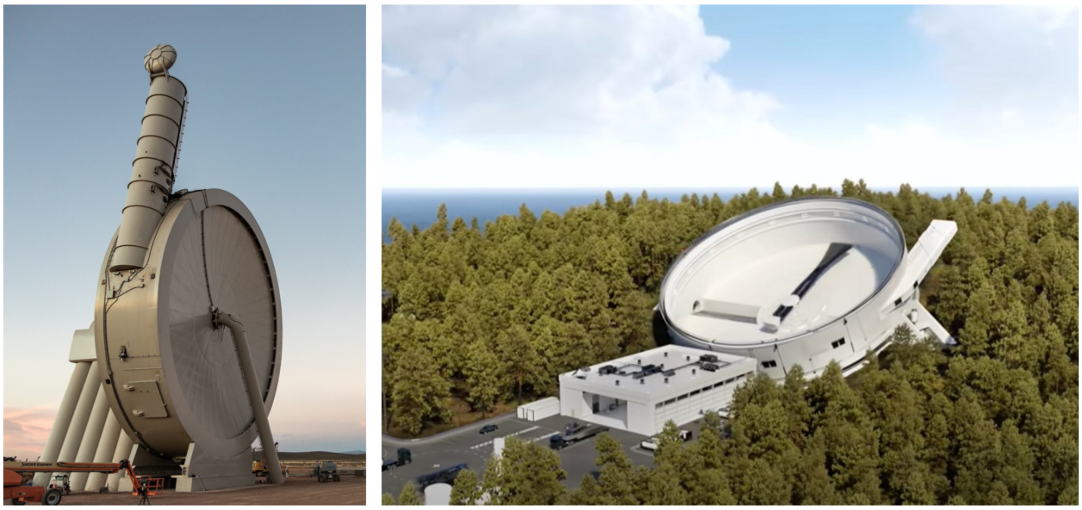

Figure 2: *Left:* The 16.5 m radius sub-orbital launcher in Spaceport America, New Mexico. Credit: SpinLaunch. *Right:* An artists impression of the 45 m radius orbital launcher at a proposed coastal site. Credit: SpinLaunch.

(a) Show that the satellites launched in the orbital launcher must be able to endure centripetal accelerations of up to $\sim 10^4\ g$.

[3]

(b) Calculate the minimum number of revolutions per second required to release the rocket.

[2]

Super Blue Moon

12. On the evening of 30th August 2023, the UK experienced a 'Blue Moon', which refers (in the modern definition) to the second full moon in a calendar month. It was also around the time of perigee and so is considered a 'supermoon' as it appears larger than normal, so was overall a 'super blue moon'. The view of it from the UK is shown in Figure 3.

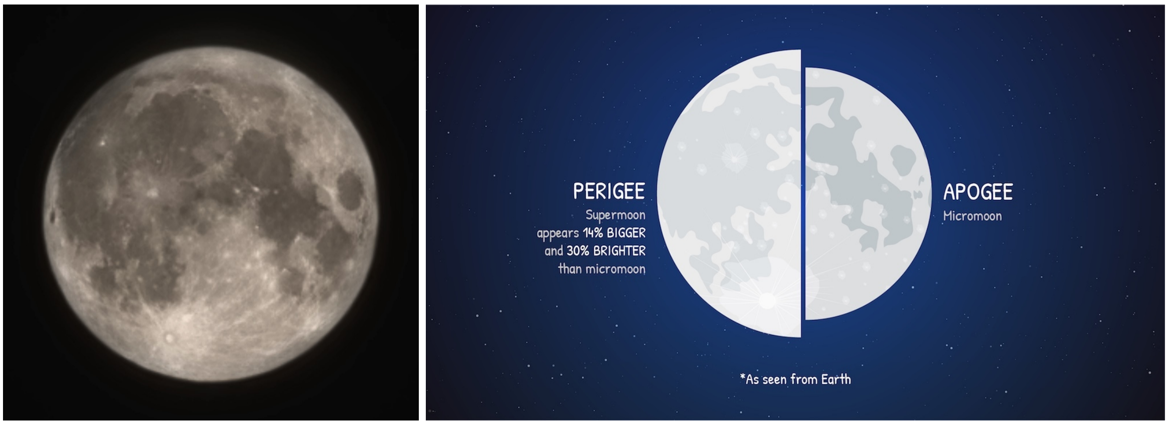

Figure 3: *Left:* The August 2023 super blue moon, as viewed from Guildford, Surrey, UK. Credit: Alex Calverley.
Right: Side-by-side comparison of a supermoon and a micromoon, showing the angular diameter of the supermoon is $\sim 14\%$ larger than the micromoon. Although the difference is pretty clear here, it is very hard for the naked eye to notice the difference when you see the supermoon on its own in the sky as you're having to compare it to a memory of what a previous full moon looked like from at least a month ago.
Credit: NASA/JPL-Caltech.

There is no formal definition of how close a full moon needs to be to the perigee to count as a supermoon, but by most definitions $\sim 25\%$ of all full moons are supermoons. Considering a supermoon that happens exactly at perigee then, relative to one that happens exactly at apogee (known as a 'micromoon'), the angular diameter of the supermoon is 14.1% larger than the micromoon (see Figure 3).

(a) Given the time between full moons is 29.53 days, how long (on average) would we have to wait for the next super blue moon? [Hint: Consider that during a year with a blue moon there will have been a total of 13 full moons in those 365 days.]

[3]

(b) Based upon the difference in angular diameter of a supermoon and micromoon, what does this suggest for the eccentricity of the Moon's orbit?

[2]

Section C: Long Answer

Each long question is worth 10 marks. Answer **either** *Qu 13* **or** *Qu 14.* **Total: 10 marks**.

Age of a Young Galaxy

13. The high-redshift galaxy GN-z11 was discovered in images from the Hubble Space Telescope (HST) in 2015 and announced in Oesch et al. (2016) as having a redshift of $z = 11.09$, making it the most distant galaxy known at the time and only overtaken recently by those discovered by the James Webb Space Telescope (JWST). Due to its extreme distance it was close to the limit of HST detectability and so it was very hard to get a good spectrum, however the superior optics of JWST has allowed Bunker et al. (2023) to get a very detailed spectrum with excellent signal-to-noise allowing a considerable number of properties of the galaxy to be measured for the first time, including a precise redshift of $z = 10.6034$, which corresponds to a luminosity distance of $d_L = 113\,150$ Mpc in their chosen cosmology. This spectrum is shown below in Figure 4.

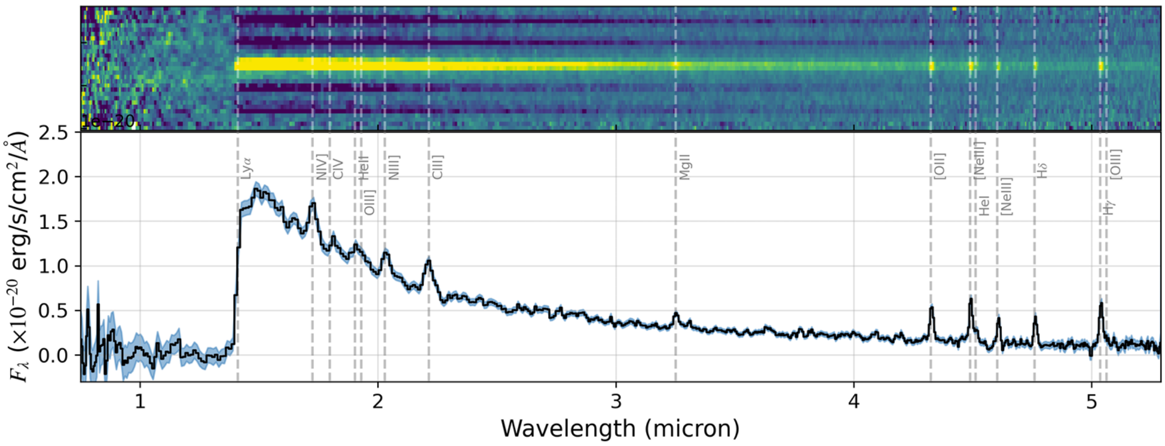

Figure 4: 2D (top) and 1D (bottom) spectra of GN-z11 using JWST. Prominent emission lines present in the spectra are marked and are are clearly seen in both the 1D and 2D spectra. A micron is 10^{-6} m. Credit: Bunker et al. (2023)

One of the advantages of a detailed spectrum is to try and determine the properties of the ultraviolet (UV) continuum (that is, the underlying shape of the spectrum, ignoring absorption and emission lines) which can be used to work out the star formation rate (SFR) and hence get an estimate for the age of the galaxy, since UV photons are mostly emitted by large, hot stars that have short lives so indicative of recent bursts of star formation.

(a) Astronomers are normally interested in the continuum UV flux at around 150 nm in the emitted (rest) frame to work out the SFR. By considering the shape of the **continuum** from rest frame 140 to 160 nm, measure the spectral flux density from the graph at $\lambda_{\text{emit}} = 150$ nm, and hence show that the absolute magnitude of this galaxy at that wavelength is $-22 < \mathcal{M} < -21$.

[5]

(b) Given the Sun has an absolute magnitude of 4.74 and assuming that all of the photons being emitted are monochromatic with $\lambda_{\text{emit}} = 150$ nm, determine the SFR of the galaxy. (Use $\mathcal{M} = -21$ if you were unable to get a value in the previous part.)

[4]

(c) The mass of the galaxy is $\sim 10^9\,M_\odot$. Assuming a constant SFR, estimate an age for the galaxy. Given the Universe is 430 million years old at this redshift, is this age possible?

[1]

Helpful formulae:
At high redshifts, the relationship between apparent and absolute magnitudes is modified to

$$m - \mathcal{M} = 5\log\left(\frac{d_L}{10}\right) - 2.5\log(1+z)\ ,$$

where d_L is the luminosity distance determined by the value of cosmological parameters used.

The apparent magnitude is related to the spectral flux density, F_ν, in units of jansky (Jy) as

$$m = -2.5\log\left(\frac{F_\nu}{3631}\right)\ .$$

Spectral flux densities in erg s^{-1} cm^{-2} Å^{-1} (the units given in Figure 4) can be converted into Jy using the unit conversion

$$F_\nu = 3.34 \times 10^6 \lambda^2 F_\lambda\ ,$$

where λ is the **observed** wavelength in nm.

Kennicut (1998) showed that the SFR, in units of $M_\odot$ year^{-1}, can be calculated as

$$SFR = 1.08 \times 10^{-53} N_{\rm ion}\ ,$$

where $N_{\rm ion}$ is the number of ionising photons emitted per second by the galaxy.

Rings of Quaoar

14. Quaoar is a trans-Neptunian object (TNO), one of the largest members of the Kuiper belt, and likely to be recognised as a dwarf planet in the near future. Despite its small size, in 2023 it was discovered to be one of only three minor planets that has a ring system. The discovery data is shown in Figure 5.

    Rings have always been associated with the Roche limit of an object, which is the distance from an object where the difference in gravitational forces on the near and far side of any orbiting satellite would exceed the internal forces of the material, tearing it apart and generating the material for the ring. The rings of Quaoar break this convention as they are much further out; simulations show that such a ring would coalesce into a satellite within a century, so they must be being constrained by some other process. The most likely explanations are that these rings are kept from coalescing due to resonances with the period of the particles in the ring and either the period of rotation of Quaoar or the period of its largest moon, Weywot.

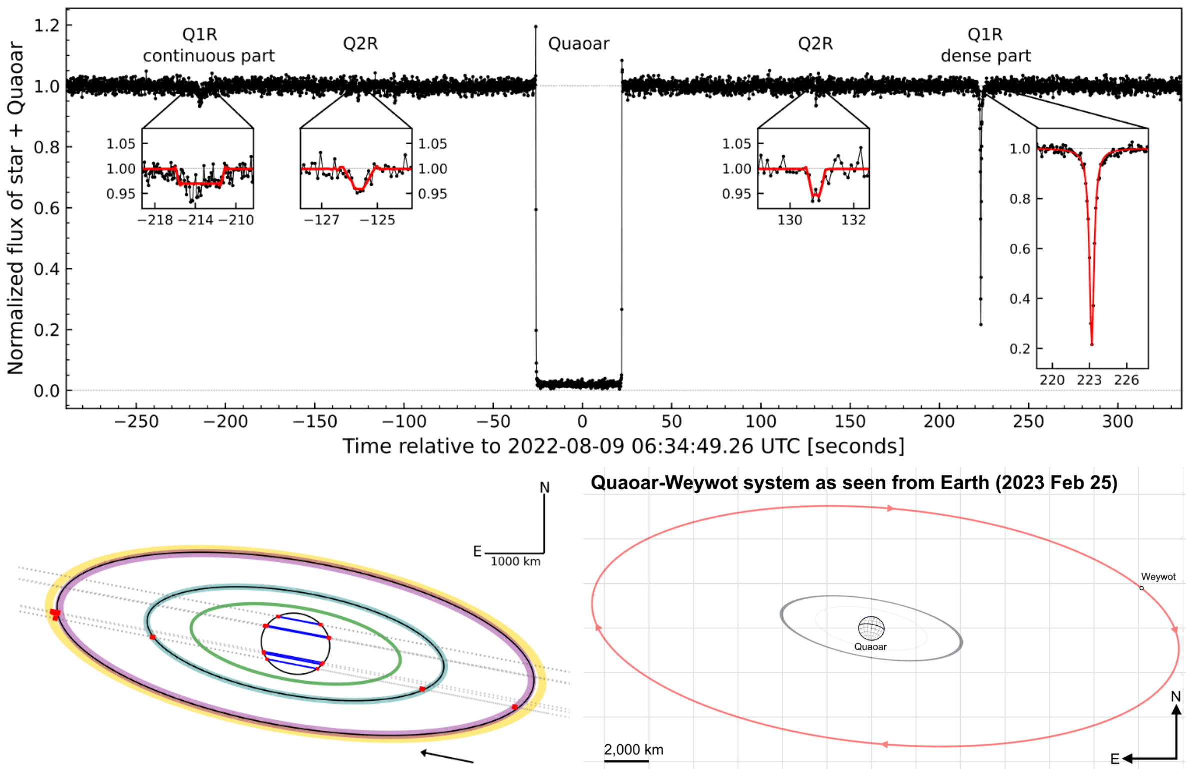

Figure 5: *Top:* The discovery of Quaoar's two rings, Q1R and Q2R, in data taken from Quaoar's occultation of a distant star (where it passed in front of the star, blocking the light). By knowing the orientation of the system and the precise times the light was blocked, the orbital parameters of the system can be determined with great accuracy. Credit: Pereira et al. (2023).
Bottom left: The central Quaoar system as derived by Pereira et al. (2023). All the slices of their observing campaign from many telescopes across the surface of the Earth during the occultation are shown as faint grey diagonal lines. The telescope slice shown in the upper panel corresponds to the bottom line. Red points show detections of the features in the spectra. The outer ring is Q1R, and is between the two resonances shown in yellow and purple. The inner ring is Q2R and is co-incident with another resonance in teal. The green ellipse represents the classical Roche limit, showing that both rings are well outside it. The orbital radius of Weywot is about three times larger than that of Q1R, and so is not in this image. Credit: Pereira et al. (2023).
Bottom right: A zoomed-out version of the Quaoar system, sufficient to show the orbit of its moon Weywot (in red) as well as the ring Q1R (dark grey). The ring Q2R is very faintly present too (but likely only visible in the electronic version of this figure). Credit: Wikipedia.

The two types of resonance involved are spin-orbit resonance (SOR), where the particles in a ring complete an integer number of orbits in the same time as an integer number of rotations of Quaoar, and mean-motion resonance (MMR), where the particles in a ring complete an integer number of orbits in the same time as an integer number of orbits of Weywot.

The Q1R ring is found at an orbital radius of 4057 km from the centre of Quaoar, shepherded between the $1:3$ SOR at 4200 km (meaning Quaoar spins 3 times in every orbit by a particle at that distance) and the $6:1$ MMR at 4020 km (meaning particles at that distance orbit 6 times whilst Weywot orbits once), as shown in Figure 5.

(a) Given Weywot has a period of 12.4311 days, calculate the rotational period of Quaoar, giving your answer in hours.

[3]

(b) Q2R has a radius of 2520 km, which is very close to another SOR of the form $a : b$. Determine a and b, given that they are single digit integers with $a < b$.

[3]

Classically, the Roche limit of this system is given by the formula

$$d = \alpha R_Q \left(\frac{\rho_Q}{\rho_{\text{ring}}} \right)^{1/3} ,$$

where R_Q is the radius of Quaoar, ρ_Q is the density of Quaoar, and ρ_{ring} is the density of the material that makes up the ring. The parameter α can vary from $2^{1/3} \approx 1.26$ for very rigid solids up to 2.44 for fluids. Tiscareno et al. (2013) found that for rings around minor planets they are best modelled as

$$\alpha = \left(\frac{4\pi}{\gamma} \right)^{1/3} ,$$

where γ is a factor describing particle shape, and that for rings like these $\gamma = 1.6$ is most appropriate (so $\alpha \approx 2.0$).

(c) Taking the density of the ring material to be $\rho_{\text{ring}} = 0.40$ g cm^{-3}, similar to the smallest inner Saturnian moons, calculate the Roche limit for the Quaoar system and hence verify that Q1R and Q2R are outside it.
Note: you do **not** need to take any measurements from Figure 5.

[4]

END OF PAPER

Questions proposed by:
Dr Alex Calverley (Surbiton High School)
Sofia Vasieva (University of Cambridge)
Josh Brown (University of Cambridge)
James Kennedy (University of Oxford)