British Astronomy and Astrophysics Olympiad 2023-2024

Astro Round 1 Section 1

Tuesday 23rd January 2024

This question paper must not be photographed or taken out of the exam room

Instructions

Time: 1 hour.

Questions: A maximum of 40 marks can be awarded for Section 1 (there are 55 marks available).

Solutions: Answers and calculations are to be written on loose paper. Students should ensure their **name** and **school** is clearly written on the **first** answer sheet and that **all** pages are numbered. A standard formula booklet with standard physical constants may be used if desired.

Clarity: Solutions must be written legibly, in black pen, and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Sitting the paper: There are two options for sitting the Astro Round 1,

1. Section 1 and Section 2 may be sat in one session of 2 hours. Section 1 should be collected in after 1 hour and only then should Section 2 be given out.
2. Section 1 and Section 2 are sat in entirely separate sessions of 1 hour each.

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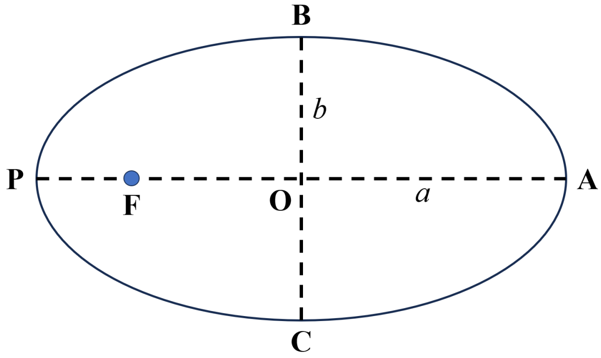
Important Constants

Constant	Symbol	Value
Speed of light	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Earth's rotation period	1 day	24 hours
Earth's orbital period	1 year	365.25 days
parsec	pc	$3.09 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	au	$1.50 \times 10^{11}\,\mathrm{m}$
Radius of the Sun	$R_{\odot}$	$6.96 \times 10^{8}\,\mathrm{m}$
Radius of the Earth	$R_{\oplus}$	$6.37 \times 10^{6}\,\mathrm{m}$
Mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	$M_{\oplus}$	$5.97 \times 10^{24}\,\mathrm{kg}$
Luminosity of the Sun	$L_{\odot}$	$3.83 \times 10^{26}\,\mathrm{W}$
Absolute magnitude of the Sun	$\mathcal{M}_{\odot}$	4.74
Hubble constant	H_0	70 km s^{-1} Mpc^{-1}
Stephan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} K^{-4}
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k_{B}	1.38×10^{-23} J K^{-1}
Permittivity of free space	ε_0	8.85×10^{-12} F m^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck's constant	h	6.63×10^{-34} J s
Elementary charge	e	1.60×10^{-19} C
Proton rest mass	m_{p}	1.67×10^{-27} kg
Electron rest mass	m_{e}	9.11×10^{-31} kg
Wien's displacement law	$\lambda_{\mathrm{max}}T$	2.90×10^{-3} m K
Avagadro's constant	N_A	6.02×10^{23} mol^{-1}

Important Formulae

You might find the diagram of an elliptical orbit below useful in solving some of the questions:



Elements of an elliptic orbit:

$a = \mathbf{OA}\ (= \mathbf{OP})$	semi-major axis
$b = \mathbf{OB}\ (= \mathbf{OC})$	semi-minor axis
$e = \sqrt{1 - \frac{b^2}{a^2}}$	eccentricity
$\mathbf{F}$	focus
$\mathbf{PF} = a(1-e)$	periapsis distance (shortest distance from **F**)
$\mathbf{AF} = a(1+e)$	apoapsis distance (longest distance from **F**)
πab	area of the ellipse

Kepler's Third Law:

$$T^2 = \frac{4\pi^2}{GM}a^3$$

Vis-Viva Equation:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Wien's Displacement Law:

$$\lambda_{\max}T = \text{constant}$$

Stephan-Boltzmann Law:

$$L = 4\pi R^2\sigma T^4$$

Brightness (Intensity):

$$b = \frac{L}{4\pi d^2}$$

Magnitudes:

$$\frac{b_1}{b_0} = 10^{-0.4(m_1 - m_0)}$$

$$m - M = 5\log\left(\frac{d}{10}\right)$$

Distance-Parallax Relation:

$$d = \frac{1}{p}$$

Rayleigh Criterion:

$$\theta = \frac{1.22\lambda}{D}$$

Redshift:

$$z = \frac{\Delta\lambda}{\lambda_{\text{emit}}} \approx \frac{v}{c}$$

Hubble's Law:

$$v = H_0 d$$

Qu 1. Short Questions

Answer as many questions as you in can in whatever order you desire. Any mark will be capped at a maximum mark of 40 from the 55 available. Note that $\mathcal{M}_\odot$ is given in the table of constants.

A. An asteroid in a circular orbit has a maximum angular separation from the Sun of $25°$ as observed from Earth. What is its orbital period in days?

[3]

B. As viewed by distant alien astronomers, what is the duration (in hours) of a transit of the Sun by the Earth, assuming they are in the plane of the Solar System and Earth appears to travel across the Sun's equator?

[3]

C. Treating a black hole as a sphere with a radius equal to the Schwarzschild radius, calculate the mass of a black hole with the same density as the Earth.

[4]

D. A simple telescope is made using two converging (convex) lenses in a cardboard tube. The distance between the lenses is $2.5x$ cm, and the combination also means images are magnified by a factor of x. For the objective lens, when an object is placed at a distance of 28.0 cm from it, it forms a real image at a distance of 11.2 cm. Use this information to find x.

[4]

E. The galaxy Markarian 231 (Mrk 231) is the closest known quasar (a galaxy with an extremely luminous core powered by accretion onto a supermassive black hole). The hydrogen-α emission line (with a rest wavelength of 656.28 nm) is observed at 683.50 nm. Given its apparent magnitude is 13.84 and assuming it was made entirely of stars like the Sun, estimate its mass.

[5]

F. Two equal mass comets C1 and C2 are in elliptical orbits around the Sun that have the same eccentricity e and with semi-major axes a_1 and a_2, respectively, where $a_1 < a_2$. When C1 is at aphelion it has the same kinetic energy as C2 has at perihelion.

(i) Derive an expression for the ratio a_2/a_1 as a function of e.

(ii) Evaluate a_2/a_1 if $e = 0.5$.

[5]

G. The star Arcturus has an apparent magnitude of -0.05, a parallax of 88.83 mas, and an angular **diameter** of 21.06 mas. At what wavelength would you expect its spectrum to peak?

[6]

H. The illuminated part of the (flat) lunar disc during a gibbous phase can be well modelled as a semicircle and a semi-ellipse. In the diagram below, the Moon is waxing, meaning the illuminated fraction is increasing from 'first quarter' (when just the right-hand side of the image was illuminated) to 'full moon' (when the whole lunar disc is illuminated).

(i) Derive an expression for the eccentricity of the semi-ellipse as a function of time t since first quarter (and valid until full moon), given the period of a full cycle of lunar phases is T.

(ii) Taking measurements from the diagram, and if $T = 29.5$ days, how many days is it until full moon? Give your answer in days to 2 s.f.

[6]

I. A student wants to investigate whether a telescope pointed at the midday Sun can be used to boil a cup of tea [Do NOT try this at home!]. They use an eyepiece with focal length 8 mm with their f/6 reflecting telescope to get a magnification of 200, and point the beam at their cup containing 200 g of water at 20 °C. Assuming only 70% of the incident solar radiation passes through the atmosphere and that the mirrors are 100% reflective, how long does the student have to wait (in minutes) until the water reaches boiling point? Assume the cup is perfectly thermally insulated and the specific heat capacity of water is 4.2 J g^{-1} °C^{-1}.

[5]

J. Due to nuclear fusion reactions in the core, a star is losing mass at a rate determined by its luminosity, however since main sequence stars follow a mass-luminosity power law of $L \propto M^{3.5}$ the luminosity will drop over time, and so will the mass loss rate. The star Nunki is the second brightest star in Sagittarius and is a very large and bright B-type main-sequence star with a current mass of $7.8\,M_\odot$ and current luminosity of $3300\,L_\odot$. Calculate how long you would have to wait until the mass of the star halved (assuming it was able to exist for that time and stay on the main sequence throughout).

[7]

K. Pulsars are highly-magnetised rotating neutron stars which emit electromagnetic radiation in beams out of their magnetic poles (see Figure 1). Since their radiation can only be observed when the beams of radiation are pointing towards Earth and the magnetic poles are not aligned with the axis of rotation, this radiation is received in pulses by observers on Earth. If the luminosity of particular pulsar that is 900 pc away is equal to $0.020\,L_{\odot}$ and its maximum apparent magnitude is 12.02, calculate the angle of aperture of the pulsar's light cones, θ. You are given that the the surface area of a spherical cap with radius r and opening angle θ is $A = 2\pi r^2(1 - \cos\frac{\theta}{2})$.

[7]

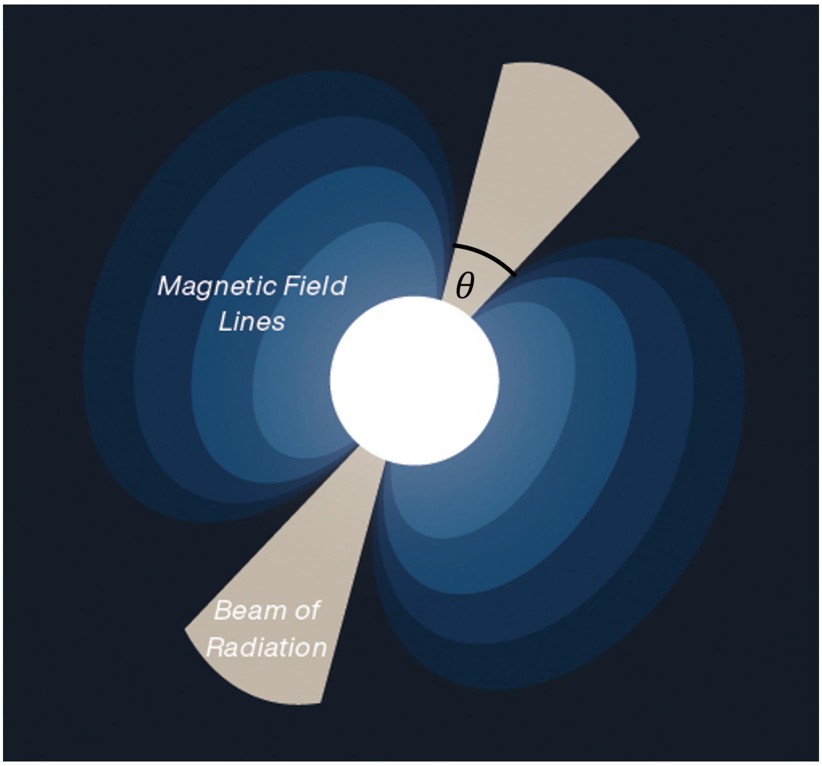


Figure 1: An artist interpretation of the parts of a pulsar, showing the magnetic field lines and the narrow beams of EM radiation with aperture angle θ. Credit: NASA/JPL-Caltech.

END OF PAPER

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