A Level Physics Online

OCR B Physics – H557

Module 5: Rise and Fall of the Clockwork Universe

You should be able to demonstrate and show your understanding of:	**Progress and understanding:**			
	1	**2**	**3**	**4**
5.1 Models and Rules				
5.1.1 Modelling Oscillations				
Harmonic motion is any motion which repeats itself				
Simple harmonic motion is harmonic motion that obeys two rules; -The force acting on the (and hence acceleration of the) oscillator is directly proportional to the displacement from the equilibrium position, x -Force acting on the (and hence acceleration of the) oscillator is always directed towards the equilibrium position - These conditions give rise to the equation $F = -kx$ Where k is the spring constant				
Another property of simple harmonic oscillators (SHOs) is that for small displacements, the time period of the pendulum does not depend on the time period. This is property is known as isochronous				
Time Period: Time for one complete oscillation				
Amplitude: The largest displacement of an oscillator from the equilibrium position				
Equilibrium Position: The position of an oscillator on which no net force is acting, for a pendulum this is at the bottom of the swing. The displacement of an SHO varies sinusoidally over time				
For a particle travelling in a circle, in a time, T, it will have an angular speed/frequency of ω = 2π/T rads^{-1}. T is the time period of the oscillation, so we can write ω = 2πf rads^{-1}				
The equations for the displacement of an SHO are given by; $x = Acos(2\pi ft) = Acos(\omega t) \quad and \quad x = Asin(2\pi ft) = Asin(\omega t)$ sin is used for when x = 0 when t = 0, cos is used for when x = non-zero when t=0. <u>Remember to use radians on your calculator for these equations</u>				

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The acceleration for a simple harmonic oscillator is given by; $$a = -\omega^2 x = -4\pi^2 f^2 x \quad (equation\ 1)$$ So at the equilibrium point, when x = 0, the acceleration is zero, the SHO is moving with constant velocity				
If we differentiate displacement wrt time, we get velocity. If we differentiate velocity wrt time, we get acceleration. Using equations; $$x = Acos(\omega t)$$ $$v = \frac{dx}{dt} = -A\omega \sin(\omega t)$$ $$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -A\omega^2 \cos(\omega t)$$ Shown graphically, it can be easily seen that the phase difference between displacement and velocity, and between velocity and acceleration is π/2 radians. So, the phase difference between displacement and acceleration is π radians				
Specific Points of the Graphs of the Above Equations Against Time: -v is zero at maximum x -a is maximum at maximum x -v is zero at maximum a -Sines and cosines have values between -1 and 1, so the maximum displacement occurs when $Acos(\omega t) = 1$, in other words, when x = A -Similarly, maximum velocity is ωA				
Another equation for acceleration can be found for an SHO if we equate Newton's second law and F=-kx $$a = -\frac{kx}{m} \quad (equation\ 2)$$ -Note that as above, acceleration is proportional to the negative displacement as it always acts towards the equilibrium position				
An iterative model can be constructed to model an SHO using equation 2 above to find the displacement and velocity at a time, t, given the mass of the oscillator, the force constant, the time interval, the initial displacement and the initial velocity; 1) $a = -\frac{kx}{m}$ to find the acceleration 2) $\Delta v = a\Delta t$ (from the definition of acceleration) 3) $v_{new} = v_{old} + \Delta v$ 4) $\Delta x = v_{avg}\Delta t = \frac{(v_{new}+v_{old})}{2}\Delta t$ 5) $x_{new} = x_{old} + \Delta x$ 6) Repeat with $x = x_{new}$ and $v_{old} = v_{new}$ from the previous interval -In this model, v_{old} is the velocity at the beginning of a given time interval, the same as the velocity at the end of the last time interval				
When an iterative calculation is plotted on a graph, the line between each successive pair of points is linear, showing that the model assumes the velocity is constant during each interval Δt. This isn't true in a real example,				

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so to improve the model Δt is made as small as possible. However, when this model runs for a long time it predicts that the amplitude of the oscillations gets bigger, not smaller. This is an example of a limitation of a simple mathematical model				
By equating equation 1 with equation 2, an expression for the frequency of an SHO can be found; 1) $a = -4\pi^2 f^2 x = -\frac{kx}{m}$ 2) $4\pi^2 f^2 = \frac{k}{m}$ 3) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ 4) $T = 2\pi\sqrt{\frac{m}{k}}$				
A similar equation can be derived specifically for a pendulum, using the fact that by similar triangles, F/T = x/L, where T is the tension and L is the length of the pendulum. This can be rearranged to get F = - Tx/L, the minus is introduced to show F is acting towards the equilibrium position. Then using the approximation that T ≈ mg, we get F = - mgx/L. Hence, a= - gx/L, which is then used as the analogous starting point in step 1) of the above derivation to get $$T = 2\pi\sqrt{\frac{l}{g}}$$ [Note: The time period is independent of the mass of the oscillator]				
Natural frequency: The frequency of a free oscillator, where no external forces act				
Free oscillations: An oscillation due to the action of a restoring force without any damping or driving forces. Free oscillations have a constant amplitude. A freely oscillating pendulum will swing at its natural frequency with the same amplitude for all time				
Forced oscillations: An oscillation driven by the action of a periodic driving force e.g. a bird flapping its wings in flight, or soldiers marching in time across a bridge				
In an SHO, potential (elastic or gravitational, depending on the system) and kinetic energy are transferred. The kinetic energy will be at its highest when velocity is highest, when the oscillator crosses the equilibrium position. The potential will be highest when the displacement of the oscillator is at a maximum. From the conservation of energy Total E = KE + PE, where PE is equal to mgh or $0.5kx^2$ depending on whether the system concerns elastic or gravitational potential energy				

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At any point in the path of an oscillator, the total energy of the system remains constant, assuming no dissipative forces act (such as friction). It just transfers between potential and kinetic				
If dissipative forces, such as friction or air resistance, act then the system is described as damped.				
Light Damping: Maximum displacement decreases after each oscillation; the period of the oscillator remains roughly constant (a grandfather clock)				
Heavy Damping: Oscillator returns to equilibrium position slower than light damping but completes less complete periods before coming to rest (imagine a pendulum suspended in golden syrup)				
Critical Damping: Oscillator stops at the equilibrium position before completing one full cycle (earthquake prevention in the foundation of buildings)				
In a damped SHO system, the maximum amplitude of the oscillations decreases over time with an exponential envelope. If you were to join the maximum amplitude points of each oscillation with a curve, that curve would fit an exponential				
Resonance: The effect produced when the driving frequency matches the natural frequency of a system, resulting in large amplitude oscillations. During resonance there is a 100% energy transfer (maximum energy transfer for a real system).				
A resonance response graph has amplitude on the y-axis and frequency on the x-axis. When the driving frequency exceeds the natural frequency, the amplitude of oscillation has a lower value than when there is no driving frequency (it dips below the value of the amplitude when f=0)				
Barton's Pendulums: This is an example of resonance. Each pendulum has a different length string attaching it to a near horizontal string above. There is one driving pendulum, a heavy weight. Different lengths change the period of the different pendulums. When the driver starts oscillating, energy from the driver is transferred to the other pendulums via the 'horizontal' string . They all oscillate with different amplitudes. The pendulum that is the same length as the driver swings with the greatest amplitude, A, the same A as the driver pendulum. Hence it has the same period and hence the same frequency as the driver pendulum.				
Damped Resonance: If the oscillator is damped, the resonance response graph would show a broader graph with a lower maximum amplitude. The peak would also move to the left for greater damping, the more a system is damped, the lower its resonant frequency				

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Uses of Resonance: ✓Tuning radios – electric signal in circuit forced by incoming radio waves ✓Microwave – molecules of water forced to vibrate by microwaves ✓Magnetic resonance in atoms – nuclei in atoms behave as magnets ×Buildings in earthquakes ×Components in engines – same rates of rotation can lead to damage ×Positive feedback in amplification systems – gives a high pitched squealing sounds, difficult to remove				